TRINITY COLLEGE

Mathematics Methods 3,4
Test 5 2017

Calculator Assumed
Normal Distribution

STUDENT'S NAME SOLUTIONS

DATE: Thursday 10 August **TIME:** 50 minutes **MARKS**: 53

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Intentional blank page.

1. (9 marks)

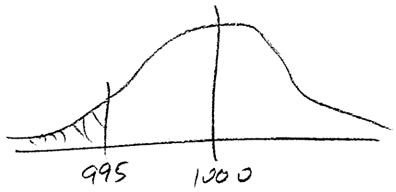
A machine fills packets with sugar. Assume that each packet is filled independently of all other packets and that the weight of the contents of each packet is normally distributed with a mean of 1 kg and a standard deviation of 2.5 g.

Each packet is weighed before sealing and if it does not contain at least 995 g of sugar it is topped up.

(a) Calculate the probability that a randomly selected packet requires topping up. [2]

$X \sim N(1000, 2.5^2)$

$P(X < 995) = 0.0228$



(b) For a day in which 1500 packets are produced, calculate the expected number of packets which require topping up. [1]

$1500 \times (1 - 0.9772)$

$= 34$

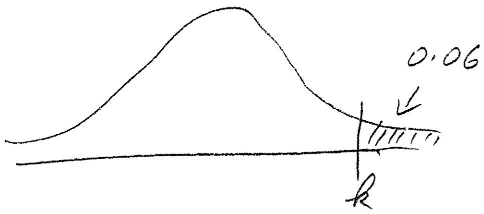
(c) Determine the probability that a randomly chosen packet weighs 1001 g. [1]

0

(d) What is the weight that is exceeded by only 6% of the packets? [2]

$P(X > k) = 0.06$

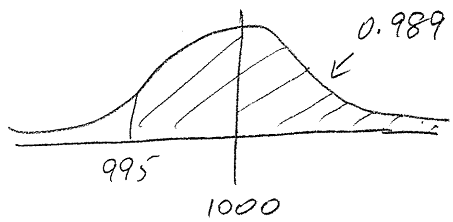
$k = 1003.9$



(e) After overhauling the machine it was claimed that the weights of the contents of the packets had a normal distribution with a mean of 1000 g and that 98.9% contained at least 995 g. What then was the standard deviation of this distribution? [3]

$P(X > 995) = 0.989$

$\sigma = 2.18$

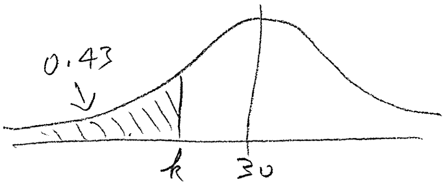


2. (8 marks)

The weight of packets of a new brand of snack food is normally distributed with a mean weight of 30 grams and a variance of 16 grams. The packets are advertised as containing 24 grams. The company making and packaging the snack food knows that it will have problems with the consumer protection group if the packets weigh less than the advertised 24 grams.

(a) Calculate the 43rd percentile and explain in words what this means. [2]

$X \sim N(30, 4^2)$

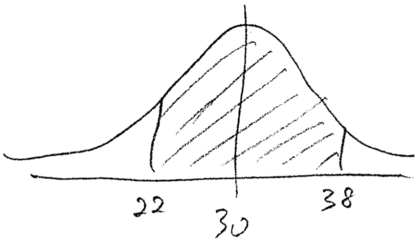


$P(X < k) = 0.43$

$k = 29.3$

43% OF WEIGHTS ≤ 29.3 g

(b) What is the probability that a randomly chosen packet of the snack food will weigh within half the variance from the mean. [2]

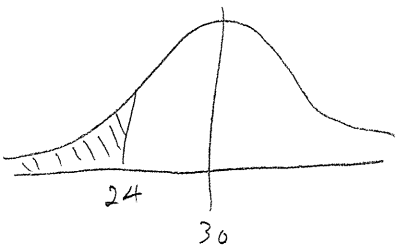


$P(22 \leq X \leq 38)$

$= 0.9545$

(c) For any new product, an attempt is made to estimate an average profit per packet. The calculation assumes that three quarters of customers who have a packet under 24 grams will complain and be given a replacement packet (that is definitely more than 24 grams) free of charge. If the company makes a profit of 5 cents a packet for all packets that are not replaced but incurs a loss of 3 cents a packet when a customer is given a replacement, calculate the average profit per packet of snack food that the company can expect. [4]

$P(X < 24) = 0.0668$



EXP PROFIT $= (1 - \frac{3}{4} \times 0.0668) \times 5 - (\frac{3}{4} \times 0.0668) \times 3$

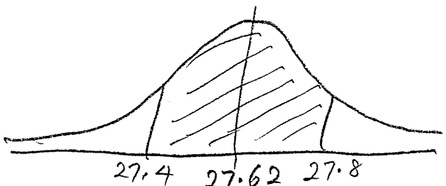
$= 4.6$ CENTS

3. (8 marks)

In Australia, size 10 shoes should be between 27.4 cm and 27.8 cm in length. A shoe manufacturer has calculated that its machinery, when set to size 10, produces shoes that are normally distributed with a mean of 27.62 cm and a standard deviation of 0.115 cm.

(a) What percentage of shoes produced will be within the size 10 range? [3]

$X \sim N(27.62,\ 0.115^2)$



$P(27.4 \le X \le 27.8)$

$= 0.9134$

$= 91.34\%$

(b) To test the operation of the machine, 10 shoes are randomly selected each hour. If two or more are found to be outside the acceptable range, the machine is serviced. What is the probability that after the next random selection the machine will be serviced? [3]

$P(\text{OUTSIDE}) = 1 - 0.9134$

$= 0.0866$

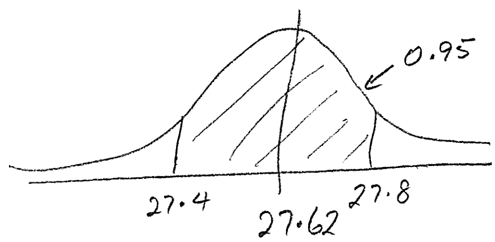
$Y \sim b(10,\ 0.0866)$

$P(Y \geq 2) = 0.2126$

(c) The manufacturer would prefer that 95% of the shoes produced be within the acceptable range. To achieve this, the machine will be adjusted to have a normal distribution with a mean of 27.6 cm and a new standard deviation. What standard deviation will be required? [2]

$Z \sim N(27.6, \sigma^2)$

$\sigma = 0.102$



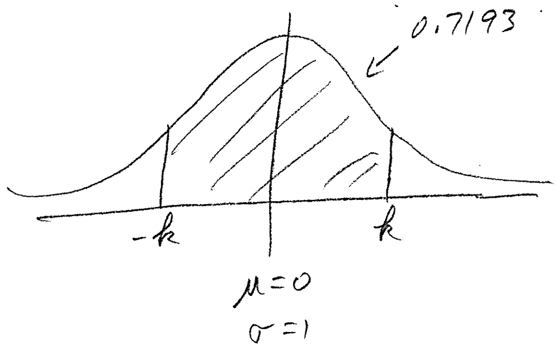
4. (6 marks)

For the Standard Normal Distribution, with mean μ, determine the value of k if

(a) $P(|\mu - k|) = 0.7193$ $\qquad Z \sim N(0, 1^2)$ [3]

$$P(0 \leq Z \leq k) = \frac{0.7193}{2}$$

$$k = 1.08$$

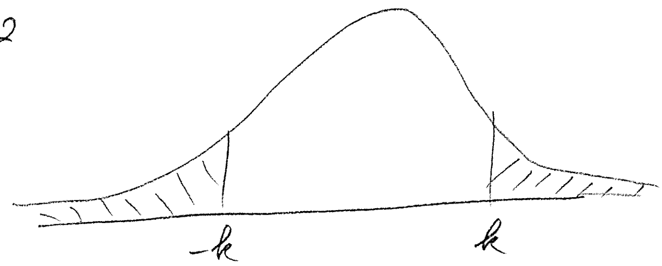


(b) $P(-k \geq z \geq k) = 0.4052$ [3]

$$P(-k \leq z \leq k) = 1 - 0.4052$$
$$= 0.5948$$

$$P(0 \leq z \leq k) = \frac{0.5948}{2}$$

$$k = 0.83$$



5. (8 marks)

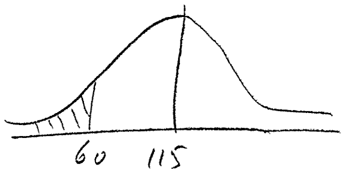
The ABC mobile phone company provides 2 hours of free access each day to its clients (the cost of the first 2 hours is built into the monthly charge). Usage beyond 2 hours a day is charged at a rate of $2 per hour or part thereof. It knows that the daily length of usage by its customers is normally distributed with a mean of 1 hour 55 minutes with a standard deviation of 20 minutes. The company has 7000 clients.

(a) Determine the probability that a randomly selected client spends:

(i) Less than 1 hour on their phone [1]

$X \sim N(115, 20^2)$

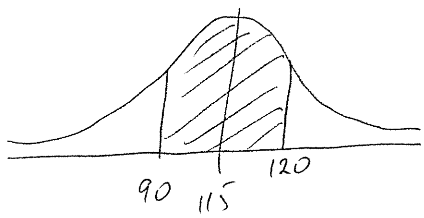
$P(X < 60) = 0.0030$



(ii) At least 90 minutes on their phone with no extra charges [2]

$P(90 \leq X \leq 120)$

$= 0.4931$

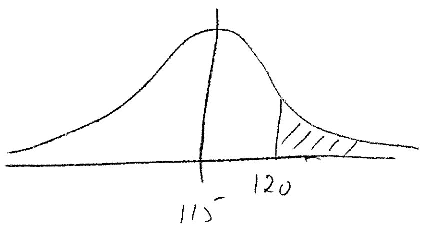


(b) Determine the number of clients expected to exceed the daily 2 hour free access. [2]

$P(X > 120) \times 7000$

$= 0.4013 \times 7000$

$= 2809$



(c) Determine the expected daily revenue collected from clients that exceed their daily free time allowance. [3]

$P(X < 120)$ ———

$P(120 < X < 180) = 0.4007$

$P(180 < X < 240) = 0.0006$

$P(X > 240)$ ———

$2805 \times 2 = \$5610$

$4 \times 4 = \$16$

TOTAL $5626

6. (14 marks)

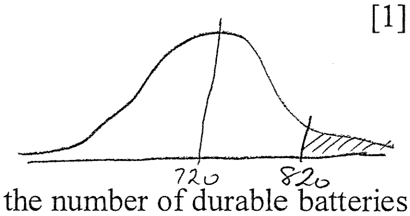
An electronics company sells two brands of batteries for personal music players, brand A and brand B. Assume that battery lives of both brands are normally distributed.

(a) A battery is described as being durable if it lasts for longer than 820 minutes. The company claims the mean battery life of Brand A is 720 minutes, with a standard deviation of 120 minutes.

(i) What proportion of Brand A batteries is durable? [1]

$X \sim N(720, 120^2)$ $\quad P(X > 820)$

$= 0.2023$



(ii) Use this result to state a Binomial distribution for the number of durable batteries in a sample size *n*. [1]

$Y \sim b(n, 0.2023)$

(iii) The company randomly samples a number of Brand A batteries. What is the minimum number needed for the sample to have a 90% chance of containing at least one durable battery? [3]

$P(Y \geq 1) = 0.9$

$P(Y = 0) = 0.1$

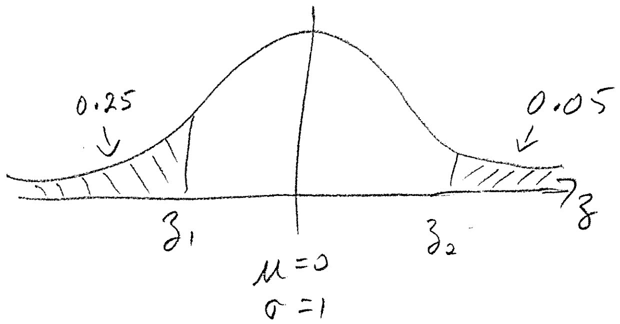
$\binom{n}{0} 0.2023^0 (1-0.2023)^n = 0.1$

$0.7977^n = 0.1$

$n = 10.2$

ie $n = 11$

(b) A survey of customers who purchased Brand B batteries showed that 25% of customers had batteries lasting less than 720 minutes and 5% had batteries lasting over 930 minutes. Clearly show the mean and standard deviation of battery lives for Brand B are 781.1 and 90.5 respectively. [3]



$z = \frac{x - \mu}{\sigma}$

$1.645 = \frac{930 - \mu}{\sigma}$

$-0.674 = \frac{720 - \mu}{\sigma}$

$z_2 = 1.645$

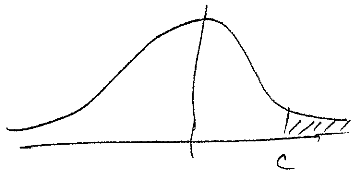
$z_1 = -0.674$

$\mu = 781.1$

$\sigma = 90.5$

(c) Determine the value of c such that the probability of a battery lasting longer than c minutes is the same for both Brand A and Brand B. [3]

$$P(A > c) = P(B > c)$$



SAME PROBABILITY

$\therefore$ SAME z SCORE

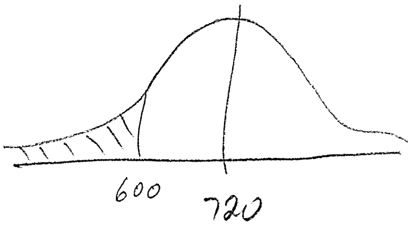
$$\frac{c - 781.1}{90.5} = \frac{c - 720}{120}$$

$$c = 968.5$$

(d) A customer rang the electronics company to complain about the length of time the battery lasted. The customer indicated that the battery only lasted 600 ~~hours~~ MINS. Was the battery the customer was complaining about more likely to be Brand A or B? Mathematically justify your answer. [3]

$$A \sim N(720, 120^2)$$

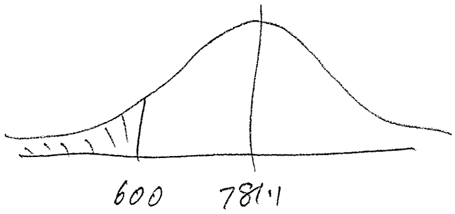
A



$$P(A < 600) = 0.1587$$

B

$$B \sim N(781.1, 90.5^2)$$



$$P(B < 600) = 0.0227$$

$\therefore$ MORE LIKELY BRAND A